COMMENT

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DETAIL FROM *SAINT PETER'S DENIAL OF CHRIST* BY GEORGES DE LA TOUR, 1650/ MUSÉE DES BEAUX ARTS NANTES/GIANNI DAGLI ORTI/THE ART ARCHIVE

Dice players in a detail from a 1650 painting by Georges de La Tour.

Value judgements

A mathematical paradox posed in a letter 300 years ago sowed the seed of economic theory by asking what money is worth, explains **George Szpiro**.

Three centuries ago, in September 1713, the Swiss mathematician Nikolaus Bernoulli wrote a letter to a fellow mathematician in France, the nobleman Pierre Rémond de Montmort. In it, Bernoulli described an innocent-sounding puzzle about a lottery. De Montmort found the problem so thought-provoking that within months, he published the letter in the second edition of his treatise *Essai d'analyse sur les jeux de hazard* ('Essay on the analysis of games of chance'). Little did the correspondents know that their exchange was the seed for the development of a fundamental concept of human decision-making, which would spawn the emergence of economics as a science.

The puzzle is about the gulf between what mathematicians expect from an uncertain event in the future, based on probability theory, and what common sense tells us to do. The reverberations of the conundrum's eventual solution 25 years later are still felt today whenever a person chooses whether to buy home insurance, a bank manager decides what interest to charge a customer, or a financier ponders whether the likely returns on a risky venture warrant investing in it.

EXPECTATION MANAGEMENT

Bernoulli asked: if person A promises to give person B one coin if he throws six points on his first toss of a die, two coins if he gets six on the second throw, four coins if he gets six on the third, eight on the fourth and so on, then what can B expect to get? Bernoulli suggested that de Montmort would find "something very interesting" in this "easy" problem.

Probability theory was then in its infancy. The idea of 'expectation' had been coined 60 years earlier, in a correspondence between two French amateur mathematicians, the philosopher Blaise Pascal in Paris and the judge Pierre de Fermat in Toulouse. The pair had concluded that the expected value of an uncertain event is computed by multiplying the potential values with the probabilities of their occurrence.

De Montmort dismissed Bernoulli's problem, writing in November 1713 that it presented no difficulty whatsoever. All one had to do was to sum the relevant series. But de Montmort had missed the point. Reproachfully, Bernoulli retorted the following February that "you would have done well to seek the solution because it would have given you an occasion to make a very curious observation".

Reframing the problem from throwing a die to the simpler case of throwing a coin, Bernoulli proposed that the payout doubles each time the heads side of a coin does not appear. For example, Peter offers to pay Paul one gold ducat if a coin lands on heads on the first throw. If the first throw is tails, and the second is heads, Paul gets two ducats. If the first two throws land on tails, and heads appears on the third throw, Paul will get four ducats. If there are three tails in a row, and then heads, the payout will be eight ducats, and so on. What can Paul expect to win?

Following Pascal and Fermat's reasoning, the win is calculated as follows. The chance of the coin landing on heads on the first throw is ½. The probability of the coin landing heads only on the second is ¼, the probability that heads will appear on the third throw, after two tails, is ⅛, and so on. The expected win is the sum of the individual payouts (1, 2, 4, 8 …) multiplied by the probabilities (½, ¼, ⅛, 1/16 …).

The result is surprising. Each product — 1 × ½, 2 × ¼, 4 × ⅛, and so on — is a half. Because the series never ends, given that there is a real, if minute, chance of a very long run of tails before the first head is thrown, infinitely many halves must be summed. Shockingly, the expected win amounts to infinity. Incredulous, de Montmort wrote: "I am not able to believe that … the advantage to Paul be infinite." Here the matter rested for 14 years.

In May 1728, writing from London, the 23-year-old mathematician Gabriel Cramer from Geneva weighed in. "Mathematicians value money in proportion to its quantity, and men of good sense in proportion to the usage that they may make of it." This was a far-ranging insight. Adding a ducat to a millionaire's account will not make him happier, Cramer reasoned. On the assumption that any amount of money beyond 2^{24} (equal to 16,777,216) gives no extra utility to its owner, he summed Bernoulli's series up to that point and obtained a finite answer. An amount of 13 ducats, Cramer claimed, was the most that one should be willing to pay to participate in the game.

But Cramer soon recognized a flaw in his own argument. An extra ducat must have some utility, be it to a pauper or to a rich person. He found a fix. The usefulness of an extra coin is never zero, but simply less than that of the previous one — as wealth increases, so does utility, but at a decreasing rate. Assuming that utility increases with the square root of wealth, Cramer recalculated the expected win to be a little over 2.9 ducats.

Bernoulli replied with a subtly different solution. The reason for the infinity paradox, he said, was not that amounts beyond some large sum give little utility to a gambler, but that the gambler disregards minute probabilities. Bernoulli set probabilities smaller than 1/32 to zero, and calculated the expected win as 2.5 ducats. Cramer's reply was contrite, even though his argument was just as legitimate.

At that point, Bernoulli turned to his younger cousin Daniel, a mathematician and physicist now known, among many other achievements, for Bernoulli's principle in fluid dynamics. Daniel's response has been lost, but we may surmise from Nikolaus's displeased answer that his cousin had sided with Cramer. The reason that Paul does not value any amount beyond 2^{24} ducats, Daniel apparently wrote, is that he fears that Peter, who must eventually pay the winnings, may not be rich enough to cover the nearly 17 million ducats if the first heads appeared only after 25 throws. Paul knows that he would never get more than that. Nikolaus dismissed his cousin's arguments, as he had Cramer's.

Frustrated, Daniel continued to work on the puzzle. Eventually, he came up with another, more elegant, solution. In July 1731, he sent a manuscript to Nikolaus in which he explained that Fermat and Pascal's original proposition — calculating the expected value as the product of the outcomes and their probabilities — was the wrong approach. If this were so, he wrote, mathematical rules would govern people's decisions and everyone would agree on the correct choices.

Refining Cramer's insight, Daniel proposed that "the value of an item must not be based on its price, but rather on the utility it yields". Instead of multiplying the potential gains of a lottery with the probabilities of their occurrence, he argued, it is the utility of each possible gain that must be multiplied by its probability. He suggested the logarithmic function as an indicator of the utility of wealth. The mean utility, converted back to its monetary value, is what a lottery is worth to the gambler. Again, Nikolaus dismissed these insights.

MEASURING RISK

Undaunted, Daniel sharpened his arguments over the next seven years. His 18-page paper on the measurement of risk was published in 1738 in *Commentaries of the Imperial Academy of Science of St Petersburg*. The solution to the 'St Petersburg paradox', as the conundrum is known, is still considered to be one of the seminal academic articles in economics. It was translated into German in 1896 and published in English in 1954 in the journal *Econometrica*.

Daniel encapsulated the probability scenario in a plot of utility versus monetary value, now known as a 'utility function' (see 'Risky business'). The curve rises — more money is always better than less — but does so at a decreasing rate, because an additional amount provides less utility to a rich person than to a pauper. The curve's diminishing gradient implies that it is always worth paying a premium to avoid a risk.

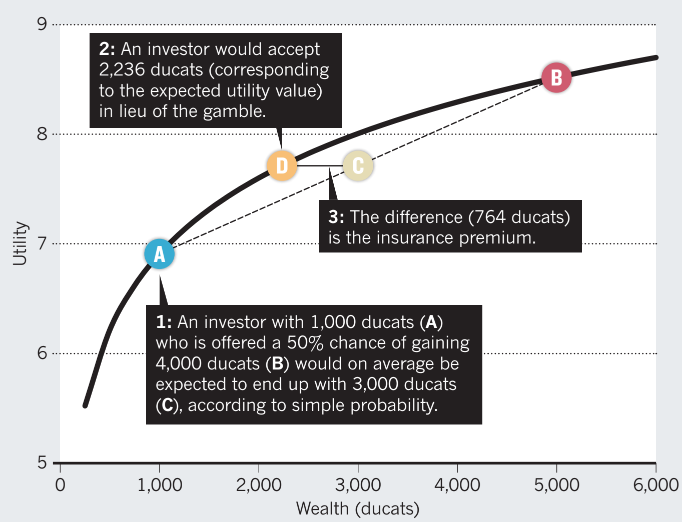
The consequences of this simple graph are enormous. Risk aversion, as expressed in the concave shape of the utility function, tells us that people prefer to receive a smaller but certain amount of money, rather than facing a risky prospect. This, in turn, implies that homeowners are willing to pay a premium to insure their belongings, that investors expect higher returns for riskier assets and that borrowing rates are higher for a jobless person taking out a loan than for a professional.

One of the first scholars to embrace Daniel's utility theory was the eminent French mathematician Pierre-Simon Laplace, who included a lecture on 'expected utility' in his 1795 lecture course on probability theory at the École Normale Supérieure in Paris. The lecture was published in 1812 in his influential *Théorie analytique des probabilités* ('Analytic theory of probability').

In the mid-nineteenth century, utility theory received support from unexpected quarters. The physician Ernst Heinrich Weber and the psychologist Gustav Theodor Fechner found that people's physical sensations of, for example, weight, exhibit the same trait as their perception of wealth: the more weight one already carries, the heavier an extra load

RISKY BUSINESS

The concave shape of the utility function (here the natural logarithm) implies that every extra ducat grants less utility to its owner than the previous one.



must be in order for it to be noticed.

Among early economists, however, Daniel Bernoulli's theory was largely ignored until the twentieth century, when mathematician John von Neumann and economist Oskar Morgenstern — in their endeavour to lift economics from "plausibility considerations" to a mathematical science — provided an axiomatic framework for utility theory and decision-making in 1944. A few years later, economist Milton Friedman and statistician Leonard Savage, puzzled by the fact that many gamblers also take out household insurance, argued that utility functions have bulges and dents. Economist Harry Markowitz adapted utility functions in 1952 such that individuals consider their current wealth as a baseline, and are either risk-averse or risk-taking depending on whether potential losses or gains are relatively small, medium or large.

At about the same time, economist Maurice Allais pointed out that utility theory does not always account for people's behaviour. Faced with lopsided choices — for example, the certainty of $1 million versus a chance of obtaining either hundreds of millions or nothing at all — people do not necessarily choose the 'rational' outcomes. Paradoxes such as these led sociologist and economist Herbert Simon to propose in the mid-1950s that humans are unable to gather all relevant information and to process it. As a result, they do not try to maximize their expected utility but, instead, set themselves more modest goals that will satisfy them.

In 1979, psychologists Daniel Kahneman and Amos Tversky developed prospect theory, which follows Daniel Bernoulli's lead but with some differences: losses hurt more than gains feel good; decisions depend on how the questions are framed; and probabilities are perceived to be smaller than they actually are, except for very small probabilities, which are perceived to be larger.

Three centuries on, Nikolaus Bernoulli's letter remains topical. Although he was not the one who provided the answer to the intriguing puzzle and, indeed, he resisted Cramer's and his cousin Daniel's explanations, it was his prompting of his friend to look deeply into the mathematics that set in motion a completely new way of thinking about risk, uncertainty and what money and wealth mean to people. ■

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ILLUSTRATION BY HARRY CAMPBELL

Mind the metaphor

Imagery can help to bridge conceptual boundaries, but it can also cause trouble — as shown by the proliferation of engineering talk in biology, argues Eleonore Pauwels.

DNA barcodes, gene-shuffling, BioBrick parts and cells as hardware: synthetic biology is saturated with metaphors. And it is not an isolated case. In 1976, evolutionary biologist Richard Dawkins coined the term 'selfish gene' to explain a DNA-centred view of evolution. Ecologists built a whole metaphorical language around the idea of the 'household of nature', including terms such as competition and colonies. Beyond the natural sciences, the father of psychoanalysis, Sigmund Freud, described the restoration of an ego damaged by neurosis as the "reclamation of flooded lands".

As a public-policy scholar, I have spent the past five years listening to synthetic biologists talk about their hopes, successes and failures. At first, I was intrigued by the pervasiveness of computing and engineering metaphors, both in conversations between scientists at the bench, and in policy discussions and public communications. Increasingly, I wanted to know what might be 'lost in translation' between these metaphors and reality. In collaboration with my colleague Andrea Loettgers, a philosopher of science at the California Institute of Technology in Pasadena, I reviewed the use of metaphors in the laboratory and in the public sphere.

We looked at several sources, including more than 1,000 synthetic-biology articles, interviews with synthetic biologists and four years of US press coverage on the subject, as well as policy reports, US congressional hearings and bioethics-commission meetings. We found that although metaphors are essential in enabling science and in communicating research to the rest of the world, their use can also mislead the public, and even scientists themselves.

With the emergence of molecular biology in the 1940s, the idea of DNA as the 'software of life' became popular in the scientific community[1,2]. Then, in the late 1990s, computer scientists, physicists and engineers were fuelled by the idea that they might ▶